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PLATE-GIRDER CONSTRUCTION

By I. HIROI, C.E.
M. Am. Soc. C.E.

FIFTH EDITION
ENTIRELY REWRITTEN AND ENLARGED

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PREFACE TO FIFTH EDITION

MORE than a quarter of a century has passed since this booklet was first published, during which time the methods of design and construction have undergone many important changes. In design, the study of dynamic action of moving loads, which on railroads have been increasing both in amount and speed on one hand and that of elastic property of the materials of construction on the other, have led to newer methods of proportioning the structures. In construction, wrought iron as bridge material has been replaced by structural steel, and the introduction of improved machines and tools have generally ad-

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vanced the workmanship. The whole subject has from time to time been more or less thoroughly discussed in books and journals.

This monograph, originally written at a time when there was scarcely any literature on the subject within easy reach, has fully served its purpose, and has been revised and corrected for the large number who still desire a simple and practical exposition of the subject, for which class the author hopes it will still prove of value.

I. H.

January, 1914.

ERRATA

Page 9, Line 8, *for* " **Other Latent Forces** " *read* " **Other Lateral Forces.**"

" 28, " 1, *for* "approximately" *read* "approximately for ordinary spans."

" 33, " 2, *for* $\frac{R=\sum_{o}^{l}Wa}{l}$ *put in* $R=\frac{\sum_{o}^{l}Wa}{l}$.

" 44, " 17, *for* "F of the flanges" *read* "F of either flange."

" 55, " 5, *for* "in order" *read* "in order to."

" 77, bottom, *for* $\frac{\sum_{o}^{x}W}{x}=\frac{\Sigma W}{l}$ *read* $\frac{\sum_{o}^{x}W}{x}=\frac{\sum_{o}^{l}W}{l}$.

" 87, line 10, *for* $\frac{16000(1.2)}{1+\frac{(65\times 12)^2}{36000(.7)^2}}=14300,$

read $\frac{16000(1.2)}{1+\frac{(65\times 12)^2}{36000(.7)^2}}=14300.$

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PLATE-GIRDER CONSTRUCTION

GENERAL STATEMENTS

1. Definition. Plate-girder is a built beam with flanges formed of angle-bars and plates or sometimes of channels and a web of one or more plates made continuous from end to end. It is a structure intermediate between a solid beam and a truss, but more allied to the former. It differs structurally from a solid rolled beam mainly in that the connection of its flanges and web is not as intimate as it is in the latter, being effected, only at intervals, by means of rivets. It again differs from an open girder in that the stresses are not all direct as they are in the latter. A comparatively high degree of rigidity possessed by plate-girders, the simplicity of their construction and the facility with which they are maintained in proper condition, are the reasons of their preference to open girders, especially for railway bridges.

2. Length. The length of a plate-girder is generally between 15 and 100 ft.; beyond the latter, economy at present makes their construction rather prohibitive. Whenever in the following the length of a plate-girder is spoken of, its end-to-end length will be meant; its relation to the effective length, by which is understood the distance from center to center of bearings, is approximately as follows:

$$\text{Total length in ft. } (l_1) = 1 + 1.02\,l$$

in which l = effective length in ft.

3. Depth. Plate-girder, as commonly built, is made of uniform depth which varies from $\frac{1}{7}$ to $\frac{1}{12}$ of its length; for smaller spans the ratio being more nearly $\frac{1}{7}$ and for the greater $\frac{1}{12}$. Theoretically, the economy of material would be attained when the amount of material in the flanges, is equal to that in the web; but such a relation is rarely adhered to in practice, owing to but slight variation in the total amount of material with the change in depth.

LOADS

4. Live-load. The moving load for which a plate-girder railway bridge is to be designed should be the heaviest engine or engines running on the line. Loadings known as Cooper's Series are very commonly used in designing railway bridges. Each of them consists of two freight-engines followed by corresponding uniform load as shown by diagram and table on page 6; the horizontal figures are weights in pounds while the vertical ones are distances in feet of wheel-loads. For short spans, heavy passenger-engines may give more unfavorable loading than these, so that a comparison should be made of the effects of both kinds of loading.

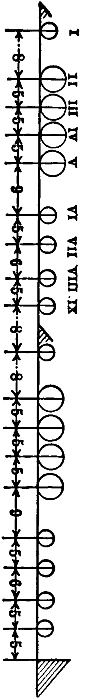
5. Impact. It is a well-known fact in bridge designing that the action of live-load is greater than that of static load of the same amount. This excess known as *impact* is estimated in various ways. A common method is by means of empirical formulas of which the following is one often used.

$$i=\frac{300}{l+300}$$

in which l=length of live-load resting on the girder, i=impact in per cent of the live-load. If we now represent by L an amount of live-load or the stress caused thereby, the equivalent static load or stress would then be

$$L(1+i).$$

6. Dead-load. The dead-load acting on a plate-girder railway bridge consists simply of the weight of the track and that of the metal in the girder. The weight of metal in a plate-girder, like that in any other kind of bridges, differs



E 30.	*E* 40.	*E* 50.
15,000	20,000	25,000
30,000	40,000	50,000
30,000	40,000	50,000
30,000	40,000	50,000
30,000	40,000	50,000
19,500	26,000	32,500
19,500	26,000	32,500
19,500	26,000	32,500
19,500	26,000	32,500
15,000	20,000	25,000
30,000	40,000	50,000
30,000	40,000	50,000
30,000	40,000	50,000
30,000	40,000	50,000
19,500	26,000	32,500
19,500	26,000	32,500
19,500	26,000	32,500
19,500	26,000	32,500
3,000 p. ft.	4,000 p. ft.	5,000 p. ft.

with the kind of loading and also, to a certain extent, with the specifications according to which the design is made. It can be roughly estimated by the use of the following empirical formulas:

Loads	Deck span	Through span
E30	$W=10(150+l_1^2)$	$W=(400+7l_1)l_1$
E50	$W=12(250+l_1^2)$	$W=(500+8l_1)l_1$

in which, W = total wt. in lbs. of metal in a single-track bridge with open floor, and

l_1 = total length of the girder in ft.

If we denote by D and L the amounts of dead- and live-loads or the stresses due to them respectively and the total load or the stress will be

$$D+L(1+i)$$

which is to be considered as a static load or the stress caused thereby.

7. Wind-pressure. The greatest wind-pressure that may act on a bridge and train is assumed to be 30 lbs. per sq. ft. at which empty cars are likely to be overturned. On the bridge alone, it may be as high as 50 lbs. per sq. ft. which is the pressure caused by a tornado. These pressures are to be assumed generally to act on the bridge horizontally on its vertical surface. But when two or more girders are set more or less closely, one behind the other, as is the case with most plate-girder bridges, the wind should have certain inclination to the horizontal, to act fully on the sides of all the girders, as shown in Fig. 1.

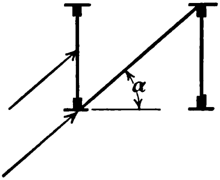


FIG. 1.

Using Hutton's formula for inclined wind-pressure we get, for the case of two girders, the following approximate relations:

α	Exposed Area
0 – 30°	2 vert. surf. of 1 girder
45°	1.8 vert. surf. of 1 girder
60°	1.3 vert. surf. of 1 girder
75°	1 vert. surf. of 1 girder

8. Other Latent Forces. Of the lateral forces developed by the rolling load, that due to the side-wise motion of the latter, owing to the slack of gauge, sets up vibration in the structure. This force would strain the braces and their connections at irregular intervals; but its amount is not possible of exact determination. Experience has shown that its effect is sufficiently provided for by assuming about $\frac{1}{10}$ of the wheel-loads covering one brace length to be active in the brace.

Centrifugal force is developed when a bridge lies on a curve. Its amount is calculated from the following formula:

$$\text{Centrif. force} = \frac{Wv^2}{32.2r},$$

in which W = moving load on the bridge;
v = velocity of the load in ft. per sec.;
r = radius of curvature of the track in ft.

The action of centrifugal force on a bridge is like that of wind-pressure, differing from the latter only in so far as the tracks or girders when located on a curve, are generally so inclined as to meet the effect of the overturning moment.

9. Longitudinal Thrust of Moving Load. This is caused by moving load when the latter is braked while running on the bridge. The greatest amount of such force would be

$$\phi W$$

in which ϕ = coef. of friction between the wheels and rails, being equal to about 0.2.

W = braked wheel-loads resting on the girder.

While it is important to know the possibility of existence of such a force, the latter is rarely taken into consideration in proportioning the girder, its effect being considered to be covered by the factor of safety.

STRESSES

10. Nature of the Stresses. While in a parallel-chord truss it is not difficult to see that bending moments produce two parallel stresses which are resisted by the chords and that the shearing forces cause in the web members stresses acting axially in them, the case is otherwise with a plate-girder. In the latter, the stresses are more complicated in that the web is continuous from end to end. We may, indeed, assume, for reasons to be explained further on, that shearing forces are resisted by the webs alone, as if the latter were web members of a truss put closely together. Such an assumption evidently simplifies the computation of the stresses and the proportioning of the parts of a girder, and is generally correct enough for all practical purposes. In designing a girder rationally, however, it is necessary to see how far such an assumption is borne out in reality, in order to know the extent of reliability to be attached to the formulas based thereon. In doing so, we shall try to view the matter in the simplest manner possible.

11. Analysis of the Stresses. It is a fundamental assumption in the common theory of flexure that, in a beam subjected to bending, a plane cross-section remains a plane through and after the bending, making the elongation or compression of the fibers and the stress causing it, proportional to their distance from the neutral axis of the beam. A study of experiments made with plate-girders has shown that, under stresses to which they are ordinarily subjected when used as bridges, such an assumption is substantiated to an extent sufficient to make it the basis for calculation of eir strength. We shall therefore, in e following, consider plate-girder as solid beam having the same dimenns.

In a beam, then, supported at both ds, and subjected to the action of rtical forces, it is evident that at any its vertical sections, by the virtue of e bending stress, that part of the beam ove the neutral axis is subjected to npression, and that below to tension, th of which stresses attain maximum lues at the outermost fibers of the am, and decrease to zero at the utral axis. This intensity of the stress any point is at once obtained from the ll-known equation of flexure:

$$\frac{M}{I}y = f, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

, the bending moment M, divided by moment of inertia I of the section of beam, multiplied by the distance y of

---

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beam
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the point from the neutral axis, gives the intensity f of the stress at that point.

Now the bending moment M increases from the end toward the middle of the beam, and with it evidently the intensity f of the horizontal stress also; so that f varies not only in vertical direction on both sides of the neutral axis, but also in the direction of the length of the beam. Let x x and $x'x'$ be two sections of a beam very close to each other, and N N the neutral axis. Then the variation of the value of f in both sections may be

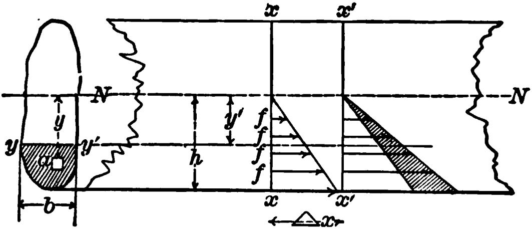

FIG. 2.

iangles *with apices in* nd the variation *in the* ion between *these two* lifferences of the areas s shown shaded in the ase of horizontal stress o another produces at ayer a force tending to yer next above it, and diminished toward the e this shearing force, increasing at every maximum intensity. lled the *longitudinal* at once obtained from us let f' be the cor- of f in section $x'\ x'$; M′ be the bending o sections $x\ x$ and $x'\ x'$ a, an infinitely small t y from the neutral

ontal stresses acting in tion lying between the nt h from the neutral

and

axis in

$$\sum_{y'}^{h} f$$

The longitudin y′ between the fore equal to

$$\sum_{y'}^{h} f'\ a -$$

Substituting in the exp of f and f' given by obtain

$$\sum_{y'}^{h} f'\ a - \sum_{y'}^{h} f\ a = ($$

Since the area on wh shear is acting is equa is the breadth of the axis, and the layer $y'\ y'$ distant y' from the axis in $x\ x$ and $x'\ x'$ are respectively:

$$\sum_{y'}^{h} f\,a, \qquad \sum_{y'}^{h} f'\,a.$$

The longitudinal shear in the layer $y'\ y'$ between the two sections is therefore equal to

$$\sum_{y'}^{h} f'\,a - \sum_{y'}^{h} f\,a.$$

Substituting in the expression, the values of f and f' given by equation (1) we obtain

$$\sum_{y'}^{h} f'\,a - \sum_{y'}^{h} f\,a = \left(\frac{M'-M}{I}\right)\sum_{y'}^{h} y\,a.$$

Since the area on which this horizontal shear is acting is equal to $b\ \Delta\ x$ when b is the breadth of the cross-section at the layer $y'\ y'$ and $\Delta\ x$ the distance between x and x', we obtain for the intensity of the shear

$$\left(\frac{M'-M}{1}\right)\sum_{y'}^{h} y\ a.\frac{1}{b\Delta x}. \quad . \quad . \quad (2)$$

The existence of such a stress becomes evident to any one, when, instead of a single beam, two beams be made to lie one upon another, which either by their own weight or specially applied forces

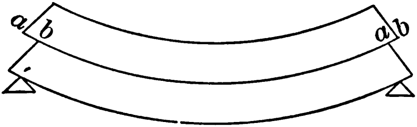

FIG. 3.

would assume a shape like that shown in Fig. (3). If they are now joined together by bolts, they would act as a single beam, and the shearing stress on these bolts would be caused by the force required to bring the two points *a* and *b* together.

The *vertical shearing action* at any section of the beam is simply the reaction due to the load at one end minus that part of the load lying between that end and the section, or

$$\left(S = R - \sum_{o}^{x} W\right), \quad . \quad . \quad . \quad (3)$$

in which S is that shear or the resultant of vertical forces at a point distant x from one end at which the upward reaction is R.

Thus at every point in the beam there are two shearing actions taking place at the same time. Imagine *a b c d* (Fig. 4) to be an infinitely small portion of the side of the beam at a point distant y' from the neutral axis. Suppose the sides of this area element be Δx and Δy and the breadth of the beam at the point to be b. There are then found two shearing stresses on this element, one vertical and the other horizontal. These two shears form two pairs of couples acting around the body, as shown by the four arrows. Let t_x. represent the intensity of the horizontal shear at this point

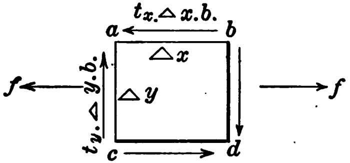

FIG. 4.

and t_y. that of the vertical. The amount of the horizontal shear is equal to

$$t_x.\Delta x.b.$$

that of the vertical shear is likewise

$$= t_y.\Delta y b.$$

In order that the body be in equilibrium, the moments of these couples must be equal, i.e.,

$$t_x.\Delta x.b.\Delta y = t_y.\Delta y.b.\Delta x$$

Consequently

$$t_x. = ty. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

showing that at every point in the beam the intensities of the vertical and horizontal shears are equal, so that we shall in the following designate them with one common letter *t*. The value of *t*, is, as already given by Eq. (2),

$$t = \frac{M' - M}{b.\Delta x\ I} \sum_{y'}^{h} a\ y. \quad . \quad . \quad . \quad (5)$$

Since

$$M = Rx - \sum_{o}^{x} W(x-d),$$

in which *d* represents the distance of loads W from the end at which the reaction is R. Likewise:

$$M' = Rx' - \sum_{o}^{x'} W(x' - d),$$

consequently

$$M' - M = R(x' - x) - \sum_{o}^{x} W(x - x),$$

so that

$$\frac{M' - M}{\Delta x} = R - \sum_{o}^{x} W,$$

and from Eq. (3) we obtain

$$\frac{M' - M}{\Delta x} = S.$$

Substituting this value of $\frac{M' - M}{\Delta x}$ in Eq. (5) we get

$$t = \frac{S}{bI} \sum_{y'}^{h} a\,y. \quad . \quad . \quad . \quad . \qquad (6)$$

This equation gives at once, at any point of the beam, the intensity of longitudinal and vertical shears.

At the neutral axis where $y' = o$ Eq. (6) becomes

$$t = \frac{S}{bI} \sum_{o}^{h} a\, y. \quad . \quad . \quad . \quad . \qquad (7)$$

There still remains to be considered the stresses acting on the sides of the element (Fig. 4); one of which is the horizontal force f, whose value has been given by Eq. (1), tending either to compress or pull asunder the two faces $a\ c$ and $b\ d$, according as it is on the upper or the lower side of the neutral axis; and the other is the direct compression on $a\ b$ due to the weight lying on it. The latter, is usually so small that it may be taken out of consideration entirely.

At the neutral axis where $f = o$, t_x and t_y are then the only stresses, and we know from mechanics that the resultant action of two equal shears at right angles to each other, exactly as t_x and t_y are, is equivalent to that of two equal and opposite stresses at right angles to each other, called the principal stresses, and making an angle of 45° with the shearing stresses. But at a distance on each side of the neutral axis, the third stress f now comes in, which evidently gives a new direction to the line of resultant stress, turning the axis of principal stresses toward itself more and more as its intensity increases.

Since for all directions and intensities of stresses there can always be found two planes at right angles to each other, and in each of which the resultant action of these stresses is normal to its surface, forming the planes of action of the principal stresses, as soon as we know the intensities and directions of these stresses, we can determine, either graphically or analytically, the direction and intensity of the principal stresses; the values of t and f at any point will give us the direction and intensities of maximum tension and compression at that point. Fig. 5 represents the appearance which the lines of principal stresses thus obtained present. The lines of maximum tension, shown dotted, cut the lines of compression at right angles. Both lines cross the neutral axis at 45° to the same, and run almost parallel to it in the middle of the beam in the neighborhood of extreme fibers. If we represent with p the intensity of stress along these curves, we have

$$p=\tfrac{1}{2}\left(f\pm\sqrt{4t^2+f^2}\right), \quad . \quad . \quad . (8)$$

giving us, at every point, two values of p, viz., compression and tension, their directions making right angles with each other.

12. Approximate Solution. In the plate-girder section of Fig. 6, let h represent the distance of centers of gravity of the upper and lower flanges, called the *effective depth* of the girder; and h_1 the

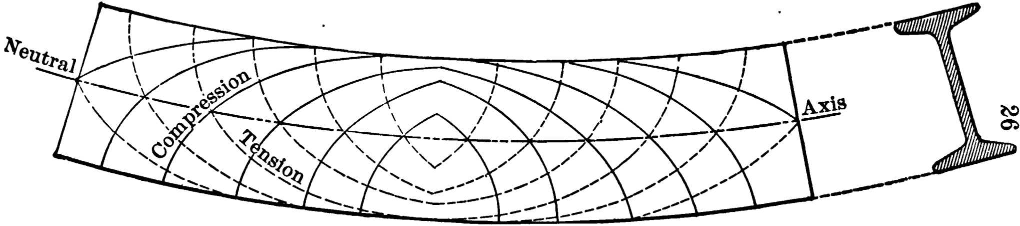

FIG. 5.

width of web-plate. Further let F denote the effective cross-sectional area of one flange and b the thickness of web-plate. Then the moment of inertia

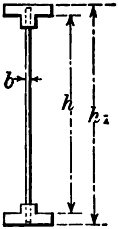

FIG. 6.

of the section with respect to the neutral axis would be approximately

$$\frac{F\,h^2}{2}+\frac{b\,h_1^3}{12}=I.$$

Now, in plate-girders as commonly built, h and h_1 are so related as can be expresed approximately by the following empirical formula:

$$h = h_1 - \left(\frac{45}{l}\right)^2,$$

in which

h and h_1 are to be expressed in inches,
l = length of girder in feet.

From the above given formula, it will be seen that h is always less than h_1 and that the difference becomes sensible only in moderate span-length. It would, therefore, be practically correct and will be on the safe side in the dimensioning of girders, to put h in place of h_1.

If we now substitute $\frac{h}{2}$ for y in Eq. (1) and introduce the above given value of I, f will give the intensity of stress in the flange and in the neighborhood of the extreme fiber of the web, such that

$$M = h\left(F + \frac{bh}{6}\right)f$$

showing that $\frac{1}{6}$ the cross-sectional area of the web is the amount which the latter contributes to making the moment of resistance. As the cross-section of web-plate is reduced by rivet-holes, it is a common practice to assume only $\frac{1}{8}$ of its area to be effective in this way, so that

$$M=h\left(F+\frac{bh}{8}\right)f. \quad . \quad . \quad . \quad (9)$$

Again in the girder section of Fig. 6, if we apply Eq. (7) and find the intensity of shear t for all points of the section and represent it graphically, it would be something like that shown by the shaded area in Fig. 7. It will at once be seen that the amount of shear sustained by the flange is comparatively small and that the increase in intensity throughout the web, is inconsiderable so that we may consider the shearing stress to be borne entirely by the web, and to be uniformly distributed over its section, i.e.,

$$\frac{S}{bh}=t. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

(t = unit shear)

Consequently at any point of the web, by dividing the shearing force at that point by the distance between the centers of gravity of flanges we obtain the amount of *shearing stress per unit of length* in its direction, which we will designate by s, making

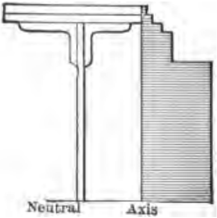

FIG. 7.

$$s = t\,b = \frac{S}{h}. \quad . \quad . \quad . \quad (11)$$

13. Flange Stress. In Eq. (9) since Ff represents the flange-stress, its amount will become known when the bending moment and the general dimensions of the girder are given.

Thus

$$Ff=\frac{M}{h}-\frac{bh}{8}f.$$

The bending moment, M is easily obtained by applying the well known equations for simple girder.

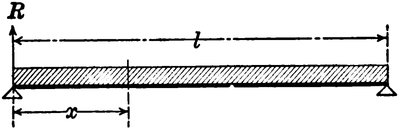

Fig. 8.

With uniformly distributed load at x (Fig. 8)

$$m=\tfrac{1}{2}wx(l-x) \quad . \quad . \quad . \quad . \quad (12)$$

in which w = uniform load per unit length.
l = effective length of the girder.

For the bending moment at the center ($x = \frac{1}{2}l$) we get

$$M = \tfrac{1}{8}wl^2.$$

The graphical representation of bending moments, due to a uniformly distributed load, is a parabola with its crown at the center of the span, as represented by the lower half of Fig. 12.

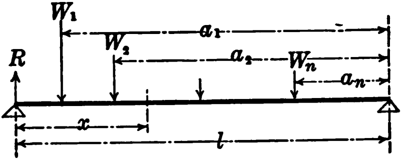

FIG. 9.

With concentrated loads (Fig. 9) the bending moment at any point x is readily obtained by applying the following equation already referred to,

$$m = Rx - \sum_{o}^{x} W(a + x - l)$$

in which

$$R = \frac{\sum_{o}^{l} Wa}{l}.$$

To find the position of a rolling load system causing maximum moment, the following points are to be kept in view:

(1) Continuous load which is uniform causes maximum moment at all points when the entire span is covered by the load, the absolute maximum being then at the middle of the span.

(2) Single concentrated load causes maximum moment at any point when the load is at the point, the absolute maximum being found at the middle of the span.

(3) When there are several concentrated loads of different amount, their center of gravity and with it, the point of maximum moment will approach the heaviest load.

(4) In the last case, the moment at any point of the girder will be a maximum

when one of the loads (generally the heaviest one) rests at the point (Fig. 10) and the rest of the loads so located as to fulfil the following condition

$$\frac{\overset{x}{\underset{o}{\Sigma}} W}{x} = \frac{\overset{l}{\underset{o}{\Sigma}} W}{l}, \quad . \quad . \quad . \quad (13)$$

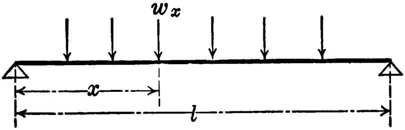

Fig. 10.

i.e., the loads on x divided by x equals the entire loads on the span divided by the span length.

The equation is generally satisfied by supposing requisite portion of W_x to be taken into $\overset{x}{\underset{o}{\Sigma}} W$.

(5) The absolute maximum moment in the last case does not occur at the center of the span, but will be found under the load which is nearest to the center of gravity of the load system and whose distance from one end of the girder equals the distance of the center of gravity from the other end, i.e. under W in Fig. 11, when $a=b$; or

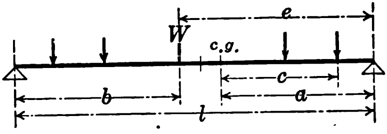

FIG. 11.

$$b=\tfrac{1}{2}(l-e+c)$$

The difference between the maximum moment at the center of the span and the absolute maximum is generally so small that the latter rarely needs to be computed.

The amount of maximum moment caused by concentrated load system at every point of the girder, when graphically shown, assumes the form of the upper portion of Fig. 12, the lower portion being, as already stated, that due to the dead-load. The sum of the ordinates at any point on both sides of the horizontal line gives the total maximum moment at that point.

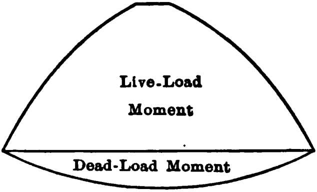

FIG. 12.

In applying the foregoing method of calculation to actual cases, the work would be considerably simplified by constructing a table giving the moments of loads with respect to successive loads and also the sums of the loads and their distances.

14. Wind-stress. The wind-pressure acting on a bridge is resisted by the lateral bracing; the latter, in plate-girders, generally forms a horizontal truss of the type of Fig. 31. In deck girders such bracing may be formed between the upper flanges only or between the lower flanges also; in the latter case, the wind-pressure acting on the structure will be resisted by the two systems of bracing. The wind-pressure acting on the train should be considered as a moving load; it is resisted by the bracing between the track-carrying flanges. The stresses in the individual members due to these pressures are obtained as in ordinary truss of the type under load. (See under numerical works.)

The wind-pressure acting on the train causes, by virtue of its overturning moment, unequal loadings on the girders, known as *transferred loads*. Figs. 13 and

14 show the cases of deck and through girders respectively, P representing the wind-pressure acting on the train and *h*, the height of its center above the lateral

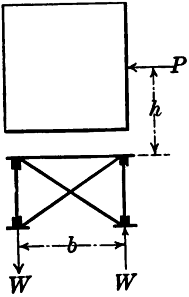

FIG. 13.

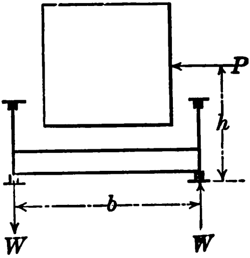

FIG. 14.

system. In either case, the lee-side girder receives an additional load of

$$W = \frac{P\,h}{b}, \quad . \quad . \quad . \quad (14)$$

while the wind-side girder is relieved of its load by the same amount. The flange-stress caused by wind-pressure is, therefore, two-fold; in one case, the flanges considered as chords of the lateral truss and in another, in its normal position, sustaining the transferred load. These actions should be duly combined as will be seen under numerical works.

15. Web-Stress. The intensity of shear in the web of a plate-girder is given by Eq. (10)

$$t=\frac{S}{b\,h},$$

and becomes at once determinate when the shearing forces and the section of the web are known. The amount S of shearing forces, is readily obtained from Eq. (3)

$$S=R-\sum_{o}^{x} W.$$

For a given position of load and hence for a given value of R, S will be maximum when $\sum_{o}^{x} W = 0$, i.e., at the head of the load or at the end of the span.

For the dead-load ($= w$ per unit length). then, we have at any point x (Fig. 8).

$$S = w\left(\frac{l}{2} - x\right), \quad . \quad . \quad . \quad (15)$$

at either end,

$$S \neq R = \frac{w\,l}{2},$$

and at the middle of the span, $S = 0$.

The maximum amount of shearing action at any point, caused by a system of moving loads, is found by the use of Eq. (3) above given. It will be seen from the equation that the greater the reaction at one end and the less the amount of the loads between that end and the point, the greater will be the amount of shear S. Generally then, the greatest amount of shear at a point due to moving loads will be found when the loads cover the greater of the two portions into which the point divides the span. With concentrated loads, the head load resting at the point as at x in Fig. 15, unless it is one considerably lighter than the succeeding one, would produce the maximum shear of the following amount between the point and the end of the span:

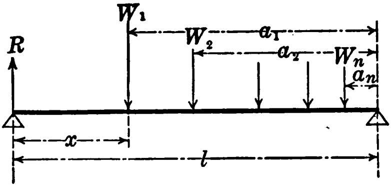

FIG. 15.

$$S = R = \frac{\Sigma W a}{l}. \qquad (16)$$

The absolute maximum shear will be found at the end of the span, the load covering the entire length or with head load at the support, the amount being equal to the reaction at that end.

Fig. 16 is the graphical representation of shear due to dead-load and also of maximum shear due to live-load at all points of the girder.

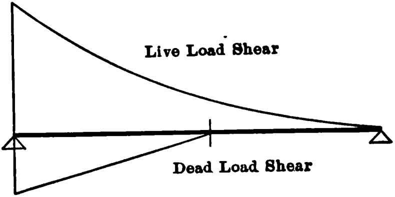

Fig. 16.

The amount of shear due to *wind-pressure* is generally so small that it may be entirely neglected.

DETAILS

16. Allowable Stress. Structural steel generally possesses an ultimate tensile strength of from 55,000 to 65,000 lbs. and an average of 60,000 lbs. per sq. in. Its elastic limit is about 36,000 lbs. per sq. in. The *unit stress* allowed for the metal against static load is in best practice, about 16,000 lbs. per sq. in.

In riveted structures, for the parts in tension, only the net cross-sectional area is effective, while for those in compression, gross section may be so considered. For slender compression-members, the above given unit stress should be reduced by a column formula; the following is one commonly used:

$$c=\frac{16000}{1+\frac{l^2}{36000r^2}},$$

in which

c = allowable compression in lbs. per sq. in.
l = length of the piece.
r = least radius of gyration of the section.

The allowable *shearing stress* in web-plate may be assumed to be 12,000 lbs. per sq. in.

Against *wind-stresses* alone or wind-stresses with other stresses combined, owing to their rare occurrence, the above given allowable stresses may be increased by 20 per cent.

17. Flanges. Knowing the maximum moment M at all points in a girder and the dimensions of the web-plate, we can at once obtain the required cross-sectional area F of the flanges from Eq. (9) as follows:

$$F = \frac{M}{16000 \times h} - \frac{b\,h}{8}, \quad . \quad . \quad . \quad (17)$$

in which dimensions are to be expressed in inches. F will be *net* section for the *lower* flange and *gross* for the *upper*.

In making up F, angles (Fig. 17*a*) or angles and plates (*b* and *c*) and sometimes channels (*d*) are used, the most common ones being the first two.

The required section is obtained by proper selection of angles and plates from any book of shapes.

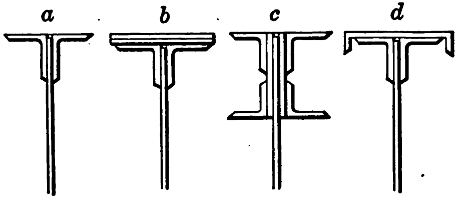

FIG. 17.

In proportioning a flange-section, it is important to bear in mind that no stress can enter the flange without first passing through the angles which alone form the connection with the web through rivets. In consequence, the flange-angles are more or less subjected to stress beside what is properly their own portion to sustain. This additional stress will evidently increase with the thickness of plates on the back. It is, therefore, advisable to use comparatively large angles consistent with the size of the girder and also to make all girders deep enough to make the packing of many plates unnecessary. It is also recommended that the vertical legs of the angles be made wide enough to take in two rows of rivets to make the connection with the web as intimate as possible without reducing the effective depth of the girder excessively.

In railway brdiges, flange-plates are avoided as much as possible, owing to the necessity of cutting into ties to clear the rivet-heads. The advantages of the use of flange-plates are the lateral stiffness given to the flanges and the water-tight covering secured for the latter.

The flange-plate which but *partly covers* the flange should be run sufficiently beyond the point, where, according to calculation, sectional area of that plate is necessary to form the required section, to catch, at least, half the number of rivets on the flange necessary to transmit the amount of stress which the plate will have to sustain. In making up the bottom flange-section, all rivet-holes which are nearly in a line must be deducted to obtain the required area. These holes should be considered to be at least $\frac{1}{8}$ in. larger than the nominal diameter of the rivets. In this connection, it is a fact worthy of notice that the majority of plate-girders having equal flanges at top and bottom, tested to destruction, failed by tension. This was evidently due to the reduction of area by rivet-holes. Rolled I-beams usually fail at the upper flanges when their webs are strong enough.

Since the upper flanges are in compression they should be *supported laterally* at intervals of not less than ten times the width of the flange, in order to make short columns of them. This is effected, in deck girders, by means of lateral bracing, and in through girders by means of gussets connected to the floor-beam as shown in Fig. 18.

18. Web. The required thickness of web-plate is obtained by substituting

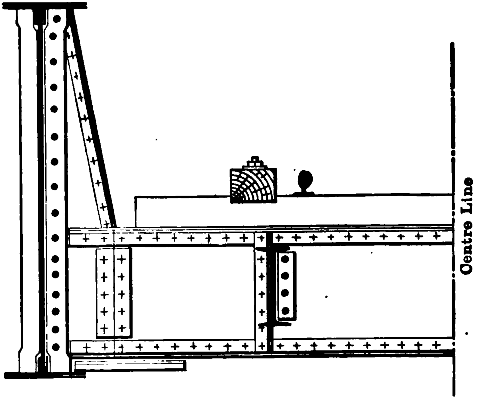

FIG. 18.

its net section for the gross one and the allowable shearing stress for t in Eq. (10). Thus

$$b = \frac{S}{12000(h_1 - vd)}, \quad . \quad . \quad (18)$$

in which v = number of rivet-holes in the vertical row, d = diameter of the rivet hole, S = maximum end reaction due to all loads. The web is generally made of uniform thickness throughout its length. The width h of a web-plate is usually made $\frac{1}{4}$ to $\frac{1}{2}$ inch less than the depth back of the flange-angles, in order to keep the rough edge of the plate below the face of the flange.

The thickness b is rarely made less than $\frac{3}{8}$ inch below which the pressure on rivets would become excessive, and oxidation by the moisture of the air may cause reduction in thickness to an undesirable extent.

It has been shown that the action of the shearing stresses at the neutral axis is equivalent to compression and tension at right angles to each other and of equal intensity, making an angle of 45° with the axis; the web is therefore still in danger of failing by flexure under this compressive stress. Consequently the web, with its thickness as already proportioned for shearing, must now be examined for its strength as a column inclined at 45°, and of the length, therefore, of h' sec. 45° (Fig. 19) and fixed

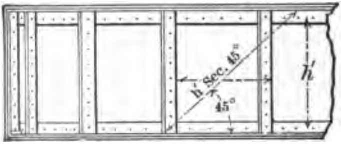

FIG. 19.

at both ends, h' being the vertical distance between the upper and lower rows of rivets in the web. For this purpose the column formula already given may be used, substituting $\frac{b^2}{12}$ for r,

$$c = \frac{16000}{1 + \frac{l^2}{3000\,b^2}}$$

in which c is the allowable compressive stress per square inch for structural steel; b the thickness of the web, and

$l = h'$ sec. 45°. In order to save the trouble of working out the formula for every case, the following table is given, from which, knowing the values of h'

$\frac{h'}{b}$	c	$\frac{h'}{b}$	c
40	7740	85	2760
42	7360	90	2500
44	7000	95	2280
46	6640	100	2080
48	6300	110	1760
50	6000	120	1520
52	5720	130	1300
54	5400	140	1140
56	5180	150	1000
58	4940	160	900
60	4700	170	800
65	4200	180	720
70	3760	190	650
75	3380	200	600
80	3040		

and b, we can at once obtain the value of c. Knowing then the amount of shear at any point where we wish to determine the strength of the web, we divide that shear by the cross-sectional area of the latter, and then compare this shearing stress per square inch thus obtained with the value of *c* in the table against the corresponding value of $\frac{h'}{b}$. If the shear per square inch is less than *c*, it shows that the web is stiff enough by itself, but if, on the contrary, *c* is less than the shear, then it shows that the web must be stiffened.

As far as the resistance against compressive stress alone is concerned, stiffeners inclined at 45° would be more effective in strengthening the web than the vertical ones, but on account of the difficulty of construction attending the former, and also on account of the direct action from the flange upon the latter when the load lies on the top of the girder, the stiffeners are always made vertical.

The *spacing of stiffeners* is a subject for experimental determination and has been more a matter of judgment than of mere calculation—the fact which our foregoing discussion of the web-stresses might suggest to any one. A rational way would be to space them, at any section, in such a manner that the web as a column having, for its length, the distance apart of stiffeners multiplied by sec. 45°, will be able to withstand the shear obtaining at the section. This method, however, makes the spacing much closer than is shown to be necessary by experience, and is, for that reason, not commonly used in practice.

A more common method is to commence putting in stiffeners where the intensity of shear begins to exceed the resistance as a column given by the above table, at intervals not exceeding the depth of the girder and gradually spacing them closer toward the ends of the girder. Some engineers specify riveting stiffeners when the thickness of the web is less than $\frac{1}{60}$ of the depth of the girder.

At the ends of the girder and at all points of concentrated loads, such as the connections of floor beams, stiffeners should be invariably put in.

Stiffeners should be put in on both sides of the web and fitted tightly between the flange-angles. In order to bring stiffeners in contact with web and vertical leg of the angles, they are either bent, as shown in Fig. 20, or fillers may be used, as in Fig. 21. The first method requires less material than the second, but requires more work in crimping the angles, and will be found economical when there are a number of similar girders to be constructed at a time. For a web less than about 3 feet in width, edge-rolled plates with length

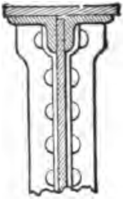

Fig. 20.

up to 30 or 40 feet are readily obtainable. For wider web, sheared plate is to be used.

19. Rivets. For rivets, softer kind of steel, known as *rivet steel*, is used in order avoid, as far as possible, the injurious effects caused by rough mechanical treatment, to which they are generally subjected in the operation of riveting. Such steel should have an ultimate tensile strength of about 52,000 lbs. per sq. in. and an elongation of over 26 per cent. The shearing stress allowed on rivets of this kind of steel should not exceed 11,000 lbs. per sq. in. and the allowable

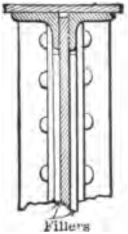

FIG. 21.

pressure on the diametral section to be under 22,000 lbs. per sq. in.

In calculating the strength of a rivet in order to be on the safe side, the *nominal* or original diameter and not the final or upset one should be taken.

Rivets are calculated to resist by shearing and bearing strengths and rarely by the tensile. The shearing strength may be simple or double. In a riveted joint such as shown in Fig. 22, the rivet resists by double-shearing and bearing strengths. Designating by d the diameter of the rivet in inches, the shearing strength in this case will be

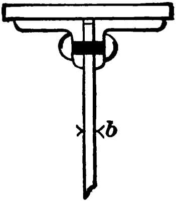

FIG. 22.

$$2\times\frac{\pi d^2}{4}\times 11000=3500\,d^2,$$

while the bearing strength equals

$$22{,}000\ bd,$$

the lesser of the two determining the strength of the joint.

Practically, rivets resist the stress by friction developed between the surfaces of pieces joined, which amounts to more than 15,000 lbs. per sq. in. of rivet section when the rivets are machine-driven.

The following table gives the shearing and bearing strengths of rivets of different diameters;

Diam. In.	Shearing Strength at 11,000 Lbs. per Sq. In.	Bearing Strength at 22,000 Lbs. per Sq. In. on Thickness of:				
	Single.	$\frac{3}{8}''$	$\frac{1}{2}''$	$\frac{5}{8}''$	$\frac{3}{4}''$	$\frac{7}{8}''$
$\frac{3}{4}$	4860	6200	8250	10300	12400	14440
$\frac{7}{8}$	6610	7220	9630	12030	14440	16840
1	8640	8250	11000	13750	16500	19250

For rivets driven in the field (field-rivets) a reduction of 10 or 20 per cent from the above given strength should be made according as they are riveted by power or hand to allow for possible imperfections in workmanship.

The *diameter of rivets* for use in a plate-girder depends on the dimensions of the latter and is generally a matter of judgment. The following diameters are those commonly used in practice:

Length of girder.	Diam. of rivets.
20– 60 ft.	$\frac{3}{4}$ in.
50– 90 "	$\frac{7}{8}$ "
80–100 "	1 "

✓ Use $\frac{7}{8}$"

It should be kept in mind that when rivet-holes are punched, rivets with diameter less than the thickness of the piece are not to be used, as the punch would be crushed when the shearing resistance becomes too great. Generally, but one diameter of rivets should be used in a girder to facilitate the shopwork.

In order to find the *number of rivets* required for a joint, it is simply necessary to divide the stress by the strength of one rivet. Any possible inequality in the distribution of the stress among all the rivets is considered to be covered by the factor of safety.

For calculating the *pitch of rivets connecting* the flanges to the web, we return to Eq. (11).

$$s=\frac{S}{h},$$

in which once more, s = amount of horizontal shear per unit length of the flange.

If we now represent by x the interval between the centers of two neighboring rivets, the horizontal shear to be resisted by one of the rivets would be sx. Denoting with r the strength of one rivet, we get

$$s\,x=r,$$

or

$$x=\frac{r}{s},$$

r being, in this case, double-shearing or bearing strength, whichever is the lesser.

In a deck girder, in which the loads rest directly on the upper flange (Fig. 23), the rivets have to transmit them to the web. If such a load is w per unit length of the flange, any one of those rivets will be subjected to diagonal stress of the amount $x\sqrt{s^2+w^2}$ so that we have

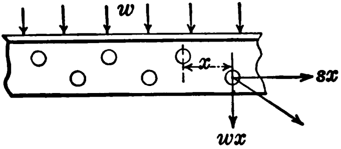

FIG. 23.

$$x\sqrt{s^2+w^2} \doteq r$$

or

$$x = \frac{r}{\sqrt{s^2+w^2}}. \quad . \quad . \quad . \quad (19)$$

Such a case sometimes occurs, also, in the lower flange of a through girder when flooring is made to rest on it—a kind of construction not to be recommended.

The rivets connecting the angles and plates in the flange are simply so spaced as to stagger with those in the vertical legs of the angles; they serve mainly to stitch the individual pieces together, so as to make them act as a single piece. When a flange-plate is wider than 12 inches, it is well to put in four lines of rivets.

The spacing of rivets is usually made alike in the upper and lower flanges to facilitate the work of fabrication. For the same reason, all rivets passing through the web, should be ranged in straight lines vertically as well as horizontally, especially when the holes are to be machine-punched.

It is a general rule in spacing rivets to limit maximum pitch in the direction of stress to 16 times the thickness of the outermost plate, and at right angle to the stress to 40 times, and minimum pitch in any direction to 3 diameters of the rivets. The minimum distance from the edge of any piece to the center of a rivet-hole should not exceed $1\frac{1}{2}$ diameters of the rivet. In an angle, rivet-head should be made to clear the side of the outstanding leg for the riveting set by *a* (Fig. 24) which is not to be less than $\frac{3}{8}$ in.

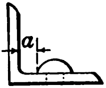

FIG. 24.

20. Splice. A plate-girder when constructed for a railway is generally completely built up and shipped in that form. Consequently when the materials with the length of the girder are readily obtainable, there is no need of splicing *the flanges*. The extreme length of angles, plates and channels depend on their lateral dimensions; a length of about 65 ft. is the maximum for the sizes commonly used for girders. There is generally no difficulty in obtaining, for ordinary sized shapes, a length of 50 or 60 ft. The extreme length of large-sized shapes is limited by the weight of the ingot. When it becomes necessary to splice flanges, the angles and plates should be spliced separately. The splicing should be made as *direct* as possible, in order to avoid detours in the transmission of stress. Fig. 25 shows a splice made in one of the flange-plates. The splice is apparently indirect; but the stress is supposed to be transmitted in

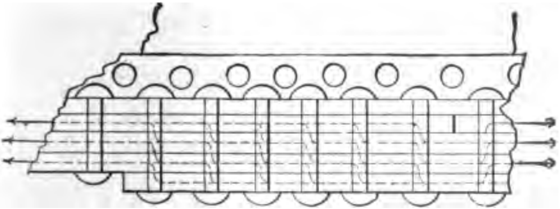

FIG. 25.

a manner shown by the dotted lines. When angles are spliced, *cover angles* should be used for the purpose; although a bar riveted on each leg may make a poor substitute. The splice plate or angle should have, at least, the same cross-sectional area as the piece joined. The number of rivets to be put in a splice on each side of the line of discontinuity should be sufficient to take the stress in the pieces connected, i.e., to be fully spliced and no transmission of stress through contact to be depended upon.

When more than two pieces are to be spliced, it is advisable to break the joints, to avoid crowding them in one place, which would necessitate the use of long rivets, beside making the joint unsightly.

A *web-plate* needs splicing, either, when plate of sufficient length is not readily obtainable or when the girder is to be built with camber (Art. 23). The limiting size of sheared plates of $\frac{3}{8}$ to 1 in. in thickness would be about 120 sq. ft.

In splicing web-plate it is to be remembered that the web has to resist bending beside shear.

If we denote by M_w that portion of the moment borne by the web, we get from Eq. (9)

$$M_w = \frac{f\,b\,h^2}{8},$$

in which f being the extreme fiber-stress equals 16,000 lbs. per sq. in.; b and h represent the thickness and width of the web-plate in inches respectively. Now, let u = resistance offered by a rivet or rivets at unit distance from the neutral axis. If d = distance of a rivet from the neutral axis, the stress in the rivet will then be ud; so that we have

$$M_w = u\,\Sigma\,d^2,$$

or

$$u = \frac{2000\,b\,h^2}{\Sigma\,d^2}.$$

Since the rivets in the splice have to

resist vertical shear beside u we get for the resultant stress,

$$\sqrt{(u\,d)^2+\left(\frac{S}{v}\right)^2},$$

in which v = number of rivets in one vertical row and S = shearing action of external forces.

If r = the strength of one rivet, which may be bearing or double-shearing in the case, n = number of rivets in a horizontal row.

$$n=\frac{1}{r}\sqrt{(u\,d)^2+\left(\frac{S}{v}\right)^2}, \quad . \quad . \quad . \quad (20)$$

In this way the necessary number and arrangement of rivets in the splice can be found; the latter, thus proportioned, generally takes the form shown in drawing on the plate.

21. Lateral Bracings. The object of lateral bracings is to resist wind pressure and vibration, and also centrifugal force when the track or roadway happens to be on a curve. In a deck girder the lateral bracing, as already stated, serves to support the upper flanges laterally. As far as the nature of stress is concerned, the lower flanges may not require any lateral support and it was customary to omit the bracing between them; but experience has shown the advisability of introducing both the upper and the lower bracings whenever possible, in order to secure lateral rigidity of the structure, which becomes more important with increased speed of trains.

The system of bracing most commonly adopted is that shown in Fig. 31. For the braces, angles are generally used; the size of these angles being determined not only by the amount of wind-stress but also by the degree of rigidity necessary for the structure. Practically, angles smaller than 3″×3″ are rarely used for the bracing. The angles are connected to the flanges by plates each large enough to take in, on its half, as many rivets as are equal in strength to the angle itself; for obvious reasons under no circumstances should less number of rivets than two be used on either side. Unless the girders are of such moderate length as to allow of being transported in whole, it would be advisable to have the connection-plates field-riveted to the flanges, as they are likely to become bent in handling and transit.

22. Cross Bracings. In deck girders, strong cross bracings are necessary at the ends of the girders to carry the wind-pressure acting on the train and girder down to the supports. At intermediate points, light ones are put in to give rigidity to the structure. Their construction is generally like that shown on the Plate.

23. Bearings. Plate-girders, with length up to about 60 ft., are generally made to rest flat on bed-plates which are more or less thickened for giving stiffness, as shown in Fig. A on the Plate. The necessary dimensions of such a bed-plate (Fig. 26) is generally calculated in the following manner:

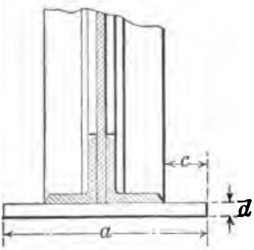

FIG. 26.

Let R = total amount of reaction at one end, including impact in lbs.
a = width of the plate in inches.
b = length of the plate in inches.
d = thickness of the plate in inches.

Assuming the allowable pressure on coping or pedestal to be 450 lbs. per sq. in. we get

$$\frac{R}{450} = a \times b.$$

For bed-plate of the same kind of material as the girder

$$d = .26\ c$$

Bed-plate for a girder of moderate span is sometimes made of cast iron. In this case

$$d = .8c$$

An objection to this kind of bearing is the tendency of the girder end to leave its seat when the girder is subjected to bending as shown exaggerated in Fig. 27. This tendency is greater the

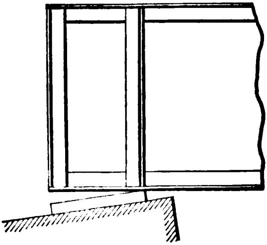
FIG. 27.

smaller the span, but the effect of unequal distribution of pressure on the bed-plate is more strongly felt in a larger span whose end reaction is naturally greater. In girders of moderate length, it is, therefore, advisable to make the bearing as shown in Fig. 28.

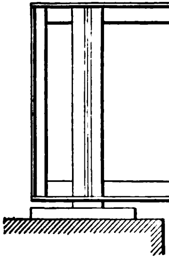

Fig. 28.

When the length of a girder is over about 70 ft. a pin-bearing, such as shown in Fig. B of the Plate, is recommended; making the shoe and bolster of steel casting.

Bed-plate and bolster should be brought

to complete contact with the stone. For this purpose, they are to be laid on a layer of cement in case of short girders, and of lead in that of longer ones. Further, they should be held down securely to the masonry by means of anchor-bolts.

In order to make it possible for the girder to expand or contract under changes of temperature, one of its ends is to be made movable. For this, in girders with length not exceeding about 80 ft. the expansion end is made sliding to an extent of

$$40' \times .000012 \times l$$

each way from its position at mean temperature by slotting the anchor-bolt holes in the flange by double that amount plus the diameter of the bolt. For a longer girder a nest of rollers, such as is used for a truss bridge, is often put in, in order to diminish the resistance to motion. When such rollers are used, it is indispensable to support the girder by pin, as already explained, to insure the uniform distribution or pressure on the rollers.

24. Camber. It has been customary to build girders with camber only for the length of more than about 60 ft., the object being nothing more than to prevent any noticeable deflection of the structure under load. A straight girder is, however, subjected to the action of centrifugal force of the passing load along the curved path formed by the deflection of the girder, the effect being more felt, the less the span length. The actual use of camber is, therefore, not merely one for appearance, but also for reducing impact to a certain extent. For giving camber to a plate-girder, a simple method is to cut each segment of the web-plate with a slight taper as shown exaggerated in Fig. 29. With the designation in Fig. 30, we at once get

$$a=b\left(1+\frac{8\,c\,h}{l^2}\right) \text{ nearly} \quad . \quad . \quad (21)$$

The upper edge of each segment is to be made longer than the lower by

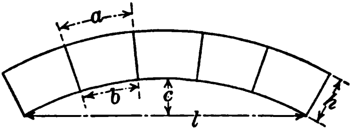

FIG. 29.

$u-b$ for producing a camber c which may range from $\frac{1}{1000}$ to $\frac{1}{2000}$ of the span length.

DECK PLATE-GIRDER BRIDGE FOR SINGLE-TRACK RAILWAY. LENGTH, 60 FT. LOAD, E40.

The following calculation and design are intended to serve as an illustration of application of the foregoing principles of construction.

According to the formula given in Art. 2, the effective length of the girder is found to be

$$=\frac{60-1}{1.02}=58 \text{ ft., nearly.}$$

Dead-load. By interpolating in formulas given in Art. 6, we get for the weight of metal in the girders,

$$W=11(200+l^2)=41{,}800 \text{ lbs.}$$

Assuming the weight of track to be 400 lbs. per ft. we get for the *total dead-load,*

Metal	41,800
Track	24,000
	65,800 lbs. = 1100 lbs. per ft. nearly

M and S Due to Dead-load. Dividing half the effective length of the girder into, say, 5 equidistant sections and numbering them as shown in Fig. 30, the moment and shear due to the *dead-load* at each section are found to be as follows:

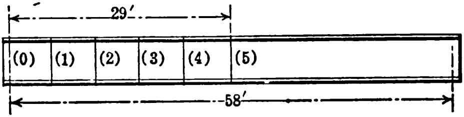

FIG. 30.

From Eqs. (12) and (15) we have at (1)

$R = 29 \times 1100 = 31,900$ lbs.

$m = 31,900 \times 5.8 - 5.8 \times 1100 \times 2.9$

$= 166,500$ ft.-lbs.

$S = 31,900 - 5.8 \times 1100 = 25,500$ lbs.

and similarly for all other points we get the following amounts:

Section.	Dead Load.	
	Moment in Ft.-lbs.	Shear in Lbs.
(0)	0	31,900
(1)	166,500	25,500
(2)	296,000	19,100
(3)	388,500	12,800
(4)	444,000	6,400
(5)	462,600	0

M Due to Live-load. For obtaining the maximum moments due to loading E40 at different points of the girder, we are to find, in the first place, the position of the wheel-loads for each point, which will satisfy the condition expressed by the equation:

$$\frac{\sum\limits_{o}^{x} W}{x} = \frac{\Sigma W}{l}.$$

Referring to the diagram and table on page 5, we see that at section (1), this condition is brought about by placing wheel-load II over the section, and for which position of the load we get for the left end reaction

$$R=\frac{40000\times 178.8+26000\times 80.8+20000\times 4.2}{58}=161000\text{ lb.}$$

and consequently for the moment

$$m=161{,}000\times 5.8=933{,}800\text{ ft.-lbs.}$$

In a similar manner, we get for all the sections the following position of loads, and the moments caused thereby:

Section.	x in Ft.	Wheel-load Over the Section.	Max. Moment in Ft.-lbs.
(1)	5.8	II	933,800
(2)	11.6	III	1,622,000
(3)	17.4	III	2,115,200
(4)	23.2	IV	2,382,100
(5)	29.0	V	2,403,500

S Due to Live-load. For causing the maximum shear at a section we find, by trial, that wheel-load II is to be placed over the section instead of I, as the latter is much the smaller of the two. For (0), then, with wheel-load II at an infinitely small distance to the right of the support we get (Art. 15)

$$S = R = \frac{40000 \times 204 + 26000 \times 104 + 20000 \times 10}{58} = 190800 \text{ lb}$$

At (2)

$$S = R - 20000 = \frac{8809600}{58} - 20000 = 131900 \text{ lbs.}$$

Similarly for all other sections we get the following shears:

Section	Max. Shear in Lbs.
(0)	190,800
(1)	161,200
(2)	131,900
(3)	103,500
(4)	77,200
(5)	54,300

Total M and S. The *total amounts of moment and shear for one girder* at the respective sections would then be the following:

Section.	Moment in Ft.-lbs.			
	Dead.	Live.	Impact.	Total.
(1)	83,300	469,800	83%	943,100
(2)	148,000	811,000	83%	1,632,200
(3)	194,300	1,057,600	83%	2,129,700
(4)	222,000	1,191,100	83%	2,401,600
(5)	231,300	1,201,800	83%	2,430,500

Section.	Shear in Pounds.			
	Dead.	Live.	Impact.	Total.
(0)	16,000	95,400	83%	190,600
(1)	12,800	80,600	85%	161,900
(2)	9,600	66,000	86%	124,400
(3)	6,400	51,800	88%	97,300
(4)	3,200	38,600	89%	71,600
(5)	0	27,200	90%	48,500

Web-section. The next step in the design of the girder is the determination of the dimensions of the web-plate. With an allowable shear of 12,000 lbs. per sq. in., the net sectional area of the web required to resist the maximum shear of 190,600 lbs. will be

$$\frac{190600}{12000} = 15.8 \text{ sq. in.}$$

If we make the width of the web-plate 72 ins. a thickness of $\frac{3}{8}$ in. would give a net section greater than that required.

Flange-section. The required *flange-sections* are now obtained in the following manner:

In Eq. (9)

$$M = f\left(F\,h + \frac{b\,h^2}{8}\right),$$

we get for h from the formula in Art. 3

$$h = 72 - \left(\frac{45}{60}\right)^2 = 71.5 \text{ in. approximately}$$

so that, with $f = 16000$ lbs. per sq. in.

$$M = \left(F + \frac{3 \times 71.5}{8 \times 8}\right) 71.5 \times 16000$$

or

$$F = \frac{M}{1144000} - 3.35.$$

With this formula, we can at once obtain the required cross-sections at different sections of the girder. Thus at (1)

$$F = \frac{943100 \times 12}{1144000} - 3.35 = 6.6 \text{ sq. in.},$$

and similarly for all other points, as follows:

Section	F in sq. in.
(1)	6.6
(2)	13.8
(3)	19.0
(4)	21.9
(5)	22.2

The flange-sections thus found, are then made up in the following manner. Upper flange:

(1) $2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' = 11.5$ sq. in.

(2) $$\left.\begin{array}{lrr} 2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' & = & 11.5 \\ 1\ \text{pl.}\ 15''\times \frac{5}{16}'' & = & 4.7 \end{array}\right\} = 16.2\ \text{sq.in.}$$

(3) $$\left.\begin{array}{lrr} 2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' & = & 11.5 \\ 1\ \text{pl.}\ 15''\times \frac{5}{16}'' & = & 4.7 \\ 1\ \text{pl.}\ 15''\times \frac{7}{16}'' & = & 6.5 \end{array}\right\} = 22.7\ \text{sq.in.}$$

(4) $$\left.\begin{array}{lrr} 2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' & = & 11.5 \\ 1\ \text{pl.}\ 15''\times \frac{5}{16}'' & = & 4.7 \\ 1\ \text{pl.}\ 15''\times \frac{7}{16}'' & = & 6.5 \end{array}\right\} = 22.7\ \text{sq.in.}$$

(5) $$\left.\begin{array}{lrr} 2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' & = & 11.5 \\ 1\ \text{pl.}\ 15''\times \frac{5}{16}'' & = & 4.7 \\ 1\ \text{pl.}\ 15''\times \frac{7}{16}'' & = & 6.5 \end{array}\right\} = 22.7\ \text{sq.in.}$$

Lower flange (rivet holes assumed to be 1 in. in diameter):

(1) $2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' = 11.5 - 2 = 9.5$sq.in

(2) $$\left.\begin{array}{lrr} 2\ \text{L}\ 6''\times 6''\times \frac{1}{2}'' & = & 11.5 - 2 \\ 1\ \text{pl.}\ 15''\times \frac{1}{2}'' & = & 7.5 - 1 \end{array}\right\} = 16.0\ \text{sq.in.}$$

$$(3)\quad \left.\begin{array}{ll} 2\text{ L }6''\times6''\times\frac{1}{2}'' & =11.5-2 \\ 2\text{ pl. }15''\times\frac{1}{2}'' & =\ \ 15-2 \end{array}\right\}\begin{array}{l}=22.5\\ \text{sq. in.}\end{array}$$

$$(4)\quad \left.\begin{array}{ll} 2\text{ L }6''\times6''\times\frac{1}{2}'' & =11.5-2 \\ 2\text{ pl. }15''\times\frac{1}{2}'' & =\ \ 15-2 \end{array}\right\}\begin{array}{l}=22.5\\ \text{sq. in.}\end{array}$$

$$(5)\quad \left.\begin{array}{ll} 2\text{ L }6''\times6''\times\frac{1}{2}'' & =11.5-2 \\ 2\text{ pl. }15''\times\frac{1}{2}'' & =\ \ 15-2 \end{array}\right\}\begin{array}{l}=22.5\\ \text{sq. in.}\end{array}$$

Wind-stresses. With the lateral bracing of Fig. 31 formed between the upper as well as the lower flanges and a wind-pressure of 30 lbs. per sq. ft. acting as shown in Fig. 32, the stresses are calculated in the following manner.

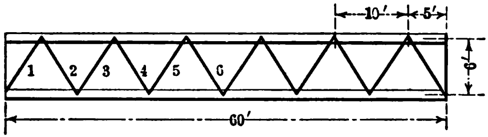

FIG. 31.

Since the pressure per panel on the upper lateral truss amounts to

On train:

$$\tfrac{1}{2}\times10'\times10'\times30\text{ lbs.}=1500\text{ lbs.}$$

On girders:

$$\tfrac{1}{2}\times 1.8\times 7'\times 10'\times 30' \text{ lbs.} = 2000 \text{ lbs.}$$

we get for the maximum upper flange-stress,

$$2500(5\tfrac{1}{2}\times 3-7\tfrac{1}{2})\frac{10}{6}=37500 \text{ lbs.}$$

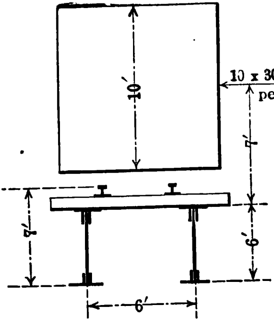

FIG. 32.

for the lower flange-stress,

$$1000(5\tfrac{1}{2}\times 3-7\tfrac{1}{2})\frac{10}{6}=15000 \text{ lbs.}$$

and for the flange-stress due to transferred load,

$$\frac{1}{8}\times\frac{300\times 7'}{6}\times 58\times 58\times\frac{12}{71.5}=24700 \text{ lbs.}$$

Stresses in Lateral Braces. In calculating the stresses in the lateral braces, the stress caused in each brace by the sidewise concussion of wheels on rails must be considered. The effect of the latter may be assumed as stated in Art. 8 to amount to

$$\frac{1}{10}\times 2\times 40000=8000 \text{ lbs.},$$

making the stress in one brace equal to

$$8000\times 1.3=10400 \text{ lbs.},$$

so that we get for the total stresses in the upper lateral braces the following amounts:

(1) $2500\times 5\tfrac{1}{2}\times 1.3+10400=28300$ lbs.

(2) $\left(1000\times 4\tfrac{1}{2}+1500\times\frac{55}{12}\right)1.3+10400=25200$ lbs.

(3) $\left(1000\times3\frac{1}{2}+1500\times\frac{45}{12}\right)1.3+10400=22300$ lbs.

(4) $\left(1000\times2\frac{1}{2}+1500\times\frac{36}{12}\right)1.3+10400=19500$ lbs.

(5) $\left(1000\times1\frac{1}{2}+1500\times\frac{28}{12}\right)1.3+10400=16900$ lbs.

(6) $\left(1000\times\ \frac{1}{2}+1500\times\frac{21}{12}\right)1.3+10400=11400$ lbs.

Since the unsupported length of any one of the braces is about $5'\times1.3=6.5$ ft., by using a $3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. angle whose least radius of gyration is 0.7 in. we get for the allowable compression

$$\frac{16000(1.2)}{1+\frac{(65\times12)^2}{36000(.7)^2}}=14300 \text{ lbs. per sq. in.}$$

The necessary cross-sections for the braces would then be

(1) $$\frac{28300}{14300}=2 \text{ sq. in.}$$

(6) $$\frac{11400}{14300}=.8 \text{ sq. in.}$$

In consideration of the alternating nature of the part of the stress and the eccentricity in connection $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles will be used for braces near to the end of the girder and $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angles for the intermediate ones.

In the lower bracing, a wind-pressure of 50 lbs. per sq. ft. causes the following stresses:

$$(1') \quad 5\tfrac{1}{2} \times \tfrac{1}{2} \times 7' \times 1.8 \times \frac{10}{2} \times 50 \text{ lbs.} \times 1.3 = 15600 \text{ lbs.}$$

$$(2') \quad 4\tfrac{1}{2} \times \tfrac{1}{2} \times 7' \times 1.8 \times \frac{10}{2} \times 50 \text{ lbs.} \times 1.3 = 9200 \text{ lbs.}$$

Combined Flange-stresses. The combined stresses in the flanges due to loading and wind-pressure are,

In the upper flange of the wind-side girder:

From load...........	355,200 lbs.
From wind...........	37,500 lbs.
From transf. load......	−24,700 lbs.
	368,000 lbs.

In the lower flange of the lee-side girder:

From load...........	355,200 lbs.
From wind...........	14,200 lbs.
From trans. load......	24,700 lbs.
	394,100 lbs.

As the greater of these combined stresses is but little more than 10 per cent in excess of the stress due to the train load, the effect of the wind-pressure, in this case, need not to be taken into consideration in proportioning the flange-sections.

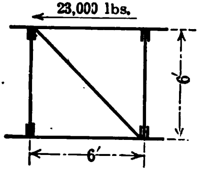

FIG. 33.

The end cross-bracing (Fig. 33) has to sustain a wind and vibration stress of $(8000+2500\times 6)1.4=32200$ lbs.

requiring

$$\frac{32200}{16000\times 1.2}=1.7 \text{ sq. in.}$$

for the net section, showing that a $5''\times 3\frac{1}{2}''\times\frac{3}{8}''$ angle is more than sufficient for the brace.

Rivet-Spacing in Flanges. The diameter of rivets used for the girder will be $\frac{7}{8}$ in. throughout. Its shearing strength is, from the table in Art. 18, found to be 6610 lbs. and the bearing value on $\frac{3}{8}$ in. to be 7220 lbs.

The spacing of rivets connecting the flanges to the web is readily found by the method explained in Art. 18.

At section (0) where total shear is 190,600 lbs. we get from Eq. (11) for the horizontal shear

$$s=\frac{190600}{71.5}=2670 \text{ lbs. per in.}$$

Since the live load rests on the upper flanges of the girder, separated by rail and ties, the wheel-load may be considered to be distributed over at least two ties or on a space of about 20 ins. (Fig. 34), making the loading 1000 lbs.

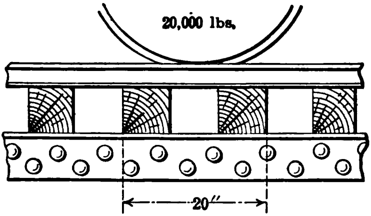

FIG. 34.

per in. Allowing in the latter 100 per cent for the impact, the resultant stress on rivets per in. of flange length would become

$$\sqrt{(2670)^2+(2000)^2}=3340 \text{ lbs. per in. horiz.}$$

so that we get for the necessary pitch of the rivets near to the end of the girder,

$$\frac{7220}{3340}=2\tfrac{1}{4} \text{ ins. nearly.}$$

Similarly, for all other points, as follows:

Section.	s	w	Result.	Pitch.
(0)	2670	2000	3340	2¼ ins.
(1)	2260	2000	3020	2½ ins.
(2)	1740	2000	2650	2¾ ins.
(3)	1360	2000	2420	3 ins.
(4)	1000	2000	2240	3¼ ins.
(5)	680	2000	2100	3½ ins.

Rivet-Spacing in Lateral Bracing. The number of rivets in the connection-plates of the lateral braces are found by dividing the stress in each brace, by single-shearing strength of one rivet. Thus in brace (1),

$$\frac{28300}{6610\times1.2}=4 \text{ rivets},$$

and for being field-riveted 10–20 per cent cent more or 5 in all.

Similarly in brace (6) we find 3 rivets to be more than sufficient.

Rivet-Spacing in the Web. For spacing the rivets in web-splice, a trial is often necessary to find the required number and arrangement of the rivets. Suppose that in the girder we are to make splices at the center and quarter points of the span. The shear at the quarter point is according to the previous calculations about 105000 lbs. As a trial we put in 15 rivets in the vertical row, as shown in Fig. 35. This would give to each rivet a stress of

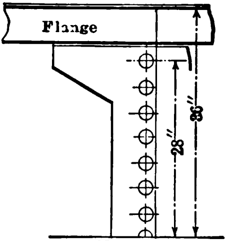

FIG. 35.

$$\frac{105000}{15}=7000 \text{ lbs.}=\frac{S}{v} \text{ (Art. 19).}$$

Let the pitch of those rivets be uniformly 4 inches; then,

$$2\,\Sigma\, d^2 = 4480 \text{ in.}^2$$

so that we get for the stress in an imaginary rivet at unit distance from the neutral axis (Art. 19).

$$u = \frac{2000 \times \frac{3}{8} \times 72 \times 72}{4480} = 870 \text{ lbs.}$$

Since the strength of rivet is, in this case, one of bearing on $\frac{3}{8}$ in. plate which is found in the table of Art. 18 to be 7220 lbs. ($= r$) we get the following number of rivets (n) required in each horizontal row:

d in In.	$\sqrt{(ud)^2 + \left(\frac{S}{r}\right)^2}$.	n
28	25,300	3.5
24	22,000	3.0
20	18,800	2.6
16	15,500	2.2
12	12,600	1.8
8	9,900	1.4
4	7,800	1.1

The arrangement of rivets in the splice, as shown in the drawing on the Plate, is made in accordance with the foregoing computations.

The splice at the center is made alike with that at the quarter point, although there is a difference in shearing stress at the two sections.

The web, being made up of four plates, each about 15 ft. in length, is made to give camber to the girder. For a camber of $\frac{l}{1200}$ each web-plate is to be tapered by the amount given by Eq. (21)

$$a=b\left(1+\frac{8\,c\,h}{l^2}\right),$$

in which

$$b=15\text{ ft.},$$
$$c=\frac{l}{1200}=0.6\text{ in.},$$
$$h=72\text{ in.},$$
$$a-b=\text{taper},$$

so that

$$a=15\left(1+\frac{8\times72\times.6}{720\times720}\right)=15\text{ ft. }0\tfrac{1}{8}\text{ in.}$$

The upper edge of each plate is, therefore, to be made $\frac{1}{8}$ in. longer than the lower.

Stiffeners. To find where stiffeners begin to be necessary, we commence at the middle of the span, where shear has been found to be 48,500 lbs. Then from Eq. (10) the intensity of shear will be

$$\frac{S}{b\,h} = \frac{48500}{\frac{3}{8} \times 72} = 1800 \text{ lbs. per sq. in.}$$

whereas, the web having for its ratio of thickness to unsupported width, $63 \div \frac{3}{8} = 168$, when considered as a column inclined at 45°, could not resist, according to the table of Art. 17, more than 800 lbs. per sq. in. It will therefore be seen that the web requires stiffening throughout its length. For the stiffeners $3\frac{1}{2} \times 5 \times \frac{3}{8}$ in. angles will be used and spaced as shown in the drawing.

End Details. Since the maximum end-reaction amounts to 190,600 lbs.

the required area for the bed-plate will be

$$\frac{190600}{450}=424 \text{ sq. in.}$$

Two kinds of the bearing are shown in the drawing; the one with pin (Fig. B) is evidently the better of the two; but for the less cost of construction, the flat one (Fig. A) is more commonly used. In the bearing of Fig. B, the shoe and bolster are to be of steel castings. In the flat bearing the thickness of the plate is determined by the formula (Art. 22):

$$d=.26\,c,$$

in which, in this case,

$$c=\frac{21-15}{2}=3 \text{ in.},$$

so that

$$d=.78 \text{ in.}$$

At the expansion-end, in the case of flat bearing, the holes in the sole-plate and in the case of pin bearing, those in the bolster are to be slotted by

$$60 \times 80^\circ \times .000012 = \tfrac{3}{4} \text{ in.}$$

making the longer diameter for $1\frac{1}{4}$ in. anchor-bolt $2\frac{1}{4}$ in.

There is no need of rollers for girders of this length.

Weight of Metal. The exact weight of metal in the girders can now be calculated, and is found to be as follows:

In flanges:	
Angles	9,200 lbs.
Plates	7,430 lbs.
In webs:	
Web-plates	10,800 lbs.
Splice-plates, etc.	1,750 lbs.
Angles	6,880 lbs.
In lateral and cross-bracings:	
Angles	2,050 lbs.
Plates	1,440 lbs.
In bearings (flat)	800 lbs.
Rivet-heads	1,800 lbs.
	42,150 lbs.

The calculated weight is found to be in excess of the estimated one; but as the difference amounts less than 1 per cent, the foregoing design needs no correction, as far as the weight is concerned.

Estimate of Cost. The cost of a plate girder bridge, like that of all metal work, depends on the price of the material, the specifications for fabrication, and, when erected, on transportation and locality. The following figures give about the average cost of construction in American bridge works.

Steel	1.40	cents	per lb.
Shop work	1.00	"	"
Engineering, etc.	0.30	"	"
	——	"	"
	2.70	"	"

42,150 lbs. at 2.7 cents per lb. =$1,138.05. To this is to be added the cost of transportation and erection; the former depends chiefly on the distance, the latter varies with the nature of the site and is generally from 0.3 to 0.6 cents or an average of about 0.5 cents per lb. of metal in girder. The total cost per lb. of the metal exclusive of transportation would then be .032 cents per lb. making estimated cost of a 60-foot girder bridge $1,348.80.

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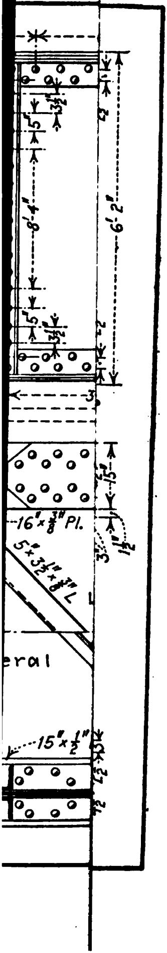

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